Chapter 2 Review

Ex: let $g(x)=2x^2+12x+18$. find a value c between 1 and 5 such that the AROC from $x=1$ to $x=5$ is equal to the IRoC of $g(x)$ at $x=c$.

$$\text{ARoC} = \frac{g(5)-g(1)}{5-1} = \frac{[2(5)^2+12(5)+18]-[2(1)^2+12(1)+18]}{4}$$

$$= \frac{50+60+18-2-12-18}{4}$$

$$= \frac{96}{4} = 24$$

IRoC at $x=c$ is $g'(c)$

We know from last class

$$g'(x) = 2(2)x+12 = 4x+12$$

So $g'(c) = 4c+12$

Now solve $4c+12=24$

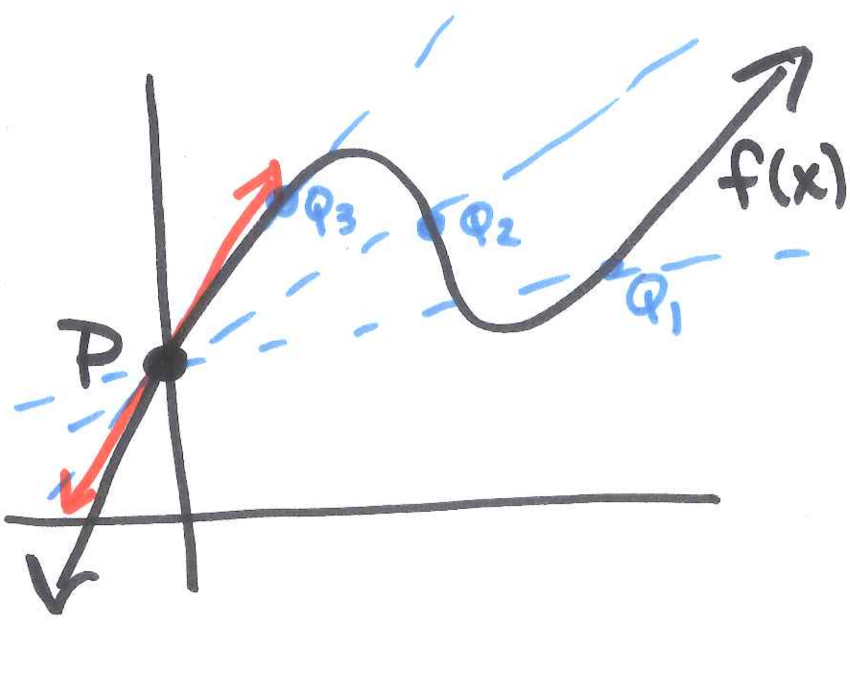
$$4c=12$$

$$\boxed{c=3}$$

The derivative of f at x_1, namely $f'(x_1)$, is the slope of the tangent line to the graph of f at the point $(x_1, f(x_1))$.

The <u>tangent line</u> to $f(x)$ at point $(x_1, f(x_1))$ is the line passing through $(x_1, f(x_1))$ with slope $f'(x_1)$, thus its equation is:

$$y = f(x_1) + f'(x_1)(x - x_1)$$



Chapter 3 - Day 1

let f be a function.

$$\lim_{x \to c} f(x) = L$$

means that as x gets close to c, through both values smaller and larger than c, but not equal to c, then the values of $f(x)$ get close to L.

<u>Note</u>: Sometimes the limit does not exist.

Ex: Consider $\lim_{x \to 3} \frac{x^2+7}{x+1}$

from the left...

x	$f(x)$
2.8	$\frac{14.84}{3.8} = 3.905$
2.9	$\frac{15.41}{3.9} = 3.951$
2.99	$\frac{15.9401}{3.99} = 3.995$

from the right...

x	$f(x)$
3.2	$\frac{17.24}{4.2} = 4.105$
3.1	$\frac{16.61}{4.1} = 4.051$
3.01	$\frac{16.0601}{4.01} = 4.005$

both sides approach 4

thus $\lim_{x \to 3} \frac{x^2+7}{x+1} = 4$

<u>Ex:</u> let $f(x) = \dfrac{x^2+7}{x+1}$. find $f(3)$.

$$f(3) = \frac{3^2+7}{3+1} = \frac{9+7}{3+1} = \frac{16}{4} = 4$$

So for functions that don't result in division by zero, then

$$\lim_{x \to c} f(x) = f(c).$$

Limit Properties:

① $\lim\limits_{x \to c} (f(x) + g(x)) = \lim\limits_{x \to c} f(x) + \lim\limits_{x \to c} g(x)$

② $\lim\limits_{x \to c} (f(x) - g(x)) = \lim\limits_{x \to c} f(x) - \lim\limits_{x \to c} g(x)$

③ $\lim\limits_{x \to c} (f(x) \cdot g(x)) = \left(\lim\limits_{x \to c} f(x)\right)\left(\lim\limits_{x \to c} g(x)\right)$

④ $\lim\limits_{x \to c} \dfrac{f(x)}{g(x)} = \dfrac{\lim\limits_{x \to c} f(x)}{\lim\limits_{x \to c} g(x)}$ if $\lim\limits_{x \to c} g(x) \neq 0$

Ex: Compute $\lim\limits_{x\to 2}\left((4x^2+x+1)\cdot(3x+7)\right)$

$$=\left(\lim_{x\to 2}(4x^2+x+1)\right)\cdot\left(\lim_{x\to 2}(3x+7)\right)$$

$$=\left(4(2^2)+2+1\right)\left(3(2)+7\right)$$

$$=19\cdot 13 = \boxed{247}$$

Ex: Suppose $\lim\limits_{x\to 1} f(x)=3$ and

$\lim\limits_{x\to 1} g(x) = -7$.

Determine $\lim\limits_{x\to 1}\left(x\cdot f(x)+\dfrac{g(x)}{x+1}\right)$

$$=\left(\lim_{x\to 1} x\right)\left(\lim_{x\to 1} f(x)\right)+\frac{\lim\limits_{x\to 1} g(x)}{\lim\limits_{x\to 1}(x+1)}$$

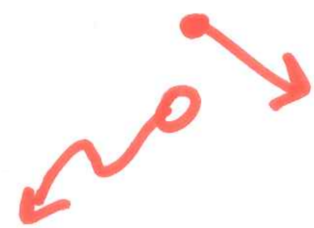
$$=1\cdot 3+\frac{-7}{2}=3-\frac{7}{2}$$

$$=\boxed{-1/2}$$

*<u>Careful</u>! You can't always substitute the value in for x

a limit can fail to exist if:

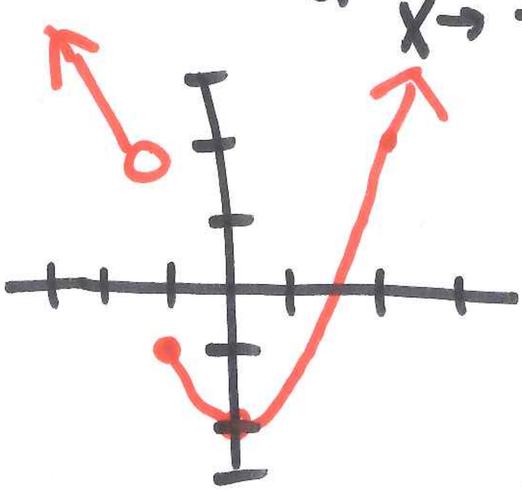
(a) The function approaches multiple values as $x \to c$



(b) the function grows without bound.

<u>Ex</u>: Consider $h(x) = \begin{cases} -x+1 & \text{if } x < -1 \\ x^2-2 & \text{if } x \geq -1 \end{cases}$

find $\lim_{x \to -1} h(x)$.



as $x \to -1$ from the left $f(x) \to 2$

as $x \to -1$ from the right $f(x) \to -1$

$\lim_{x \to -1} h(x) = \text{DNE}$

One Sided Limits

$\lim_{x \to c^-} f(x)$ is the limit from the left of c

$\lim_{x \to c^+} f(x)$ is the limit from the right of c.

Fact: $\lim_{x \to c} f(x)$ exists if and only if both one-sided limits exist and

$$\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x)$$